1 Formula Sheet

Newtons Law of Cooling/Heating:

$$\frac{dT}{dt} = k(T - T_1)$$

Useful Trig Identities:

$$\begin{aligned}
\sin^2(x) + \cos^2(x) &= 1 \\
\tan^2(x) + 1 &= \sec^2(x) \\
\sin^2(x) &= \frac{1 - \cos(2x)}{2} \\
\cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin(\cos^{-1}(x)) &= \sqrt{1 - x^2} \\
\cos(\sin^{-1}(x)) &= \sqrt{1 - x^2} \\
\sec(\tan^{-1}(x)) &= \sqrt{1 + x^2} \\
\tan(\sec^{-1}(x)) &= \begin{cases} \sqrt{x^2 - 1} & x \geq 1 \\ -\sqrt{x^2 - 1} & x \leq -1 \end{cases} \\
\sin(mx)\cos(nx) &= \frac{1}{2}[\sin(m+n)x + \sin(m-n)x] \\
\sin(mx)\sin(nx) &= -\frac{1}{2}[\cos(m+n)x - \cos(m-n)x] \\
\cos(mx)\cos(nx) &= \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]
\end{aligned}$$

Useful Derivatives:

$$\begin{aligned}
D_x \sin^{-1}(x) &= \frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \cos^{-1}(x) &= -\frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \tan^{-1}(x) &= \frac{1}{1 + x^2} \\
D_x \sec^{-1}(x) &= \frac{1}{|x|\sqrt{x^2 - 1}} \quad |x| > 1
\end{aligned}$$

Some Useful Integral Forms:

$$\int \tan u du = -\ln|\cos(u)| + C$$

$$\int \cot(u) du = \ln|\sin(u)| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$$

Note: The practice midterm may be a little longer than the actual midterm, but it reflects problems similar in difficulty to what you may encounter in the actual exam.

2 True/False

1. L'Hospital's Rule can be used to solve for the limit of any sequence $\{a_n\}$.

2. Any infinite series of all positive terms will diverge.

3. $\lim_{x\to\infty} \ln(\sqrt{x^2e^x})e^{-x^2} diverges$

4. The limit of an infinite sequence $\{a_n\} = f(n)$ is the same as the limit of the generating function $f(x)$ as x approaches infinity.

5. We can solve analytically (write down a formula/answer for) any integral of the form $\int \tan^m(x)\sec^n(x)dx$ where m,n are both any real numbers.

3 Free Response

Evaluate the following integrals:

1. $\int \frac{x^2-2x}{\sqrt{x-2}}dx$

2. $\int \frac{x+4}{x^2-9} dx$

3. $\int_1^\infty \frac{1}{(x-1)^2} dx$

Evaluate the Following Limits:

4. $\lim_{x \to 1^+} \frac{\int_1^x \sin t dt}{x-1}$

5. $\lim_{x \to 0} x^x$

6. $\lim_{n\to\infty} a_n$ where $\{a_i\} = 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...$

7. Find the limit of the corresponding series, $\sum_{i=1}^{\infty} a_i$